Exercices

Ex 1

Let $A = \begin{bmatrix} 1 & 1 \\ -2 & -1 \\ 1 & 2 \end{bmatrix}_{3\times 2}$ $B = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & -2 \end{bmatrix}_{2\times 3}$ $C = \begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 0 \\ -3 & -1 & 0 \end{bmatrix}_{3\times 3}$

Find the following if possible:

a) AB ; b) BA ; c) AC ; d) CA ; e) CB , f) BC

a) $AB = \begin{bmatrix} 3 & 0 & -4 \\ -4 & 1 & 6 \\ 5 & 1 & -6 \end{bmatrix}_{3\times 3}$

b) $BA = \begin{bmatrix} 1 & -2 \\ -2 & -3 \end{bmatrix}_{2\times 2}$

c) AC does not exist

d) $CA = \begin{bmatrix} -4 & -6 \\ -5 & -3 \\ -1 & -2 \end{bmatrix}_{3\times 2}$

e) CB does not exist

d) $BC \begin{bmatrix} 8 & 1 & -3 \\ 7 & 6 & -6 \end{bmatrix}_{2\times 3}$

Ex 2

$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 1 & -1 \end{bmatrix}_{3\times 2}$ $B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix}_{2\times 3}$ $C = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}_{2\times 2}$

$D = \begin{bmatrix} -1 & 1 \\ 4 & -3 \end{bmatrix}_{2\times 2}$ $E = \begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times 1}$

a) $-3A^t$

b) $3B - A^t$

c) E^tB

d) EE^t

e) B^tB

f) CA^t

g) D^tBE

a) $-3A^t$

$$A^t = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \end{bmatrix}_{2\times 3} \qquad -3A^t = \begin{bmatrix} -3 & -9 & -3 \\ -6 & -6 & 3 \end{bmatrix}_{2\times 3}$$

b) $3B - A^t$

$$3B\begin{bmatrix} 6 & -15 & 6 \\ -9 & 6 & 3 \end{bmatrix}_{2\times 3} - A^t\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \end{bmatrix}_{2\times 3} = 3B - A^t\begin{bmatrix} 5 & -18 & 5 \\ -11 & 4 & 4 \end{bmatrix}_{2\times 3}$$

c) E^tB

$$E^t = \begin{bmatrix} 1 & 3 \end{bmatrix}_{1\times 2} \qquad B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix}_{2\times 3}$$

$$E^tB = \begin{bmatrix} -7 & 1 & 5 \end{bmatrix}_{1\times 3}$$

d) EE^t

$$E = \begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times 1} \qquad E^t = \begin{bmatrix} 1 & 3 \end{bmatrix}_{1\times 2} \qquad EE^t = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}_{2\times 2}$$

e) B^tB

$$B^t = \begin{bmatrix} 2 & -3 \\ -5 & 2 \\ 2 & 1 \end{bmatrix}_{3\times 2} \qquad B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix}_{2\times 3} \qquad B^tB = \begin{bmatrix} 13 & -16 & 1 \\ -16 & 29 & -8 \\ 1 & -8 & 5 \end{bmatrix}_{3\times 3}$$

f) CA^t

$$C = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}_{2\times 2} \qquad A^t = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & -1 \end{bmatrix}_{2\times 3} \qquad CA^t = \begin{bmatrix} 5 & 7 & -1 \\ 5 & 15 & -5 \end{bmatrix}_{2\times 3}$$

g) $D^t BE$

$$D^t = \begin{bmatrix} -1 & 4 \\ 1 & -3 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 2 & -5 & 2 \\ -3 & 2 & 1 \end{bmatrix}_{2\times 3} \qquad E = \begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times 1}$$

Does not exist — nb of columns in B ≠ nb of rows in E

Ex3

Find a, b, c and d in that

$$* \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

$$\begin{array}{ll} (\times 2) & (\times 2) \\ a - 2c = 1 & b - 2d = 0 \\ 2a - 3c = 3 & 2b - 3d = 2 \end{array}$$

$$\begin{array}{ll} \Rightarrow 2a - 4c = 2 & 2b - 4d = 0 \\ 2a - 3c = 3 & 2b - 3d = 2 \\ -c = -1 & -d = -2 \\ \boxed{c = 1} & \boxed{d = 2} \end{array}$$

$$\begin{array}{ll} 2a - 4 = 2 & 2b - 8 = 0 \\ 2a = 6 & 2b = 8 \\ \boxed{a = 3} & \boxed{b = 4} \end{array}$$

* $\begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 7 & -7 \end{bmatrix}$

. $a + 3c = 6$ $\quad b + 3d = -5$

$a + 4c = 7$ $\quad b + 4d = -7$

$-c = -1$ $\quad -d = 2$

$\boxed{c = 1}$ $\quad \boxed{d = -2}$

$a + 3 = 6$ $\quad b - 6 = -5$

$a = 6 - 3$ $\quad b = -5 + 6$

$\boxed{a = 3}$ $\quad \boxed{b = 1}$

* $\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$

. $a + 2 = 1$ $\quad b - 3 = -2$ $\quad c + 0 = 3$ $\quad d + 1 = -4$

$\boxed{a = -1}$ $\quad \boxed{b = 1}$ $\quad \boxed{c = 3}$ $\quad \boxed{d = -5}$

Ex 4 (A)

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

Find A^{-1} if possible

$$[A|I_n] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 1 & 4 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - 2R_1 \to R_2$
$R_3 - R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & -2 & -1 & -1 & 0 & 1 \end{array}\right]$$

$R_3 - \frac{2}{3} R_2 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -2 & -2 & 1 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$R_2 + 6R_3 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & 0 & 6 & -3 & 6 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$R_1 - 9R_3 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & 6 & -9 \\ 0 & -3 & 0 & 0 & -3 & 6 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$R_1 + \frac{2}{3} R_2 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 4 & -5 \\ 0 & -3 & 0 & 0 & -3 & 6 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & -\frac{2}{3} & 1 \end{array}\right]$$

$R_2 / -3 \to R_2$
$R_3 \times 3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 4 & -5 \\ 0 & 1 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 1 & 2 & 3 \end{array}\right]$$

$$A^{-1}$$

Ex 4 (B)

Let A $\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$ Find A^{-1} if possible.

$R_2 - 2R_1 \to R_2$
$R_3 - R_1 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right]$$

$-R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 3 & -2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right]$$

$R_2 + 2R_3 \to R_2$
$R_1 - 3R_3 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 3 \\ 0 & 3 & 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right]$$

$R_2/3 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 3 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right]$$

A^{-1}

Ex 5

Let A

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & -3 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - R_1 \to R_2$
$R_3 - 2R_1 \to R_3$
$R_4 - R_1 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -2 & -1 & 1 & 0 & 0 \\ 0 & -3 & -3 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 - 3R_2 \to R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -2 & -1 & 1 & 0 & 0 \\ 0 & 0 & -9 & 4 & 1 & -3 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_4 + \frac{1}{9} R_3 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -2 & -1 & 1 & 0 & 0 \\ 0 & 0 & -9 & 4 & 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 4/9 & -8/9 & -1/3 & 1/9 & 1 \end{array}\right]$$

$R_3 - 9R_4 \to R_3$
$R_2 + \frac{9}{2} R_4 \to R_2$
$R_1 - \frac{9}{2} R_4 \to R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 5 & 3/2 & -1/2 & -9/2 \\ 0 & -1 & 2 & 0 & -5 & -1/2 & 1/2 & 9/2 \\ 0 & 0 & -9 & 0 & 9 & 0 & 0 & -9 \\ 0 & 0 & 0 & 4/9 & -8/9 & -1/3 & 1/9 & 1 \end{array}\right]$$

$R_2 + \frac{2}{9} R_3 \to R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 5 & 3/2 & -1/2 & -9/2 \\ 0 & -1 & 0 & 0 & -3 & -1/2 & 1/2 & 5/2 \\ 0 & 0 & -9 & 0 & 9 & 0 & 0 & -9 \\ 0 & 0 & 0 & 4/9 & -8/9 & -1/3 & 1/9 & 1 \end{array}\right]$$

$R_1 + 2R_2 \to R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{4} \\ 0 & -1 & 0 & 0 & -3 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & -9 & 0 & 9 & 0 & 0 & -9 \\ 0 & 0 & 0 & \frac{4}{9} & -\frac{8}{9} & -\frac{1}{3} & \frac{1}{9} & 1 \end{array}\right]$$

$-R_2 \to R_2$

$R_3 / -9 \to R_3$

$R_4 \times \frac{9}{4} \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 3 & \frac{1}{2} & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -2 & -\frac{3}{4} & \frac{1}{4} & \frac{9}{4} \end{array}\right]$$

A^{-1}